PROTON-PROTON SCATTERING AT VERY HIGH ENERGIES

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ABSTRACT
A theoretical investigation is carried out on the properties of proton-proton elastic scattering at large energies. It is assumed that both the total and elastic cross sections are rising with energy. Theoretical results are compared to experimental data obtained by Amaldi, Biancandelli, Bosio, Matthiae, Allabys, Bartel, Block, Cocconi, Diddins, Dobinson, Litt and Wetherall.

ASSUMPTIONS AND RESULTS
Proton-proton elastic scattering has been measured at the CERN Intersecting Storage Rings and the results show that both the total and elastic cross sections are rising with energy (Amaldi et al., 1973). These results are consistent with earlier studies based on cosmic-ray data (Yodh, Pal, Trefill; 1972). In this note we consider amplitudes that give rise to increasing cross sections. We assume, first of all, that the effective number of partial waves contributing is

\[ L = C \sqrt{S} \text{ (ins) where } 0 < S < 1 \]

(Prosaev, 1963). This leads to the following bound on the total cross section (Mickens, 1970):

\[ \sigma_T^2(0) \leq 16\pi C^2 \text{ (ins)^2} \]

Our second assumption is that \( \sigma_T^2(S) \) has the same energy behavior as the above bound, i.e.,

\[ \sigma_T^2(S) \sim C_L^L (\Lambda N S)^{2\gamma}, \quad C_L \leq 16\pi C^2 \]

where \( \gamma \) is a constant. This shows that while \( \sigma_T^2(S) \) and \( \sigma_T^2(0) \) may grow like \( (\text{ins})^{2\gamma} \), their difference can increase at most like \( \text{ins} \).

We have the following lower bound on the elastic cross section (Mickens, 1970),

\[ \sigma_{el}(S) \geq \left( \frac{C_1}{16\pi C^2} \right) (\Lambda N S)^{2\gamma} \]

Since \( \sigma_{el}(S) \) is bounded from above by eq. (1), we see that the elastic cross section must have the following energy behavior,

\[ \sigma_{el}(S) \sim C_1 (\Lambda N S)^{2\gamma} \]

i.e., the elastic cross section increases at the same rate as the total cross section.

We have the following bound on the phase of the forward scattering amplitude (Mickens, 1970),

\[ |\rho(S)| = \left| \frac{\text{Re} F(S,0)}{\text{Im} F(S,0)} \right| \leq \left[ \frac{C_1}{C^2} \right]^\frac{1}{2} \]

If we denote by \( \rho(S) \) and \( \bar{\rho}(S) \) the phases for the particle, antiparticle processes, one can show that

\[ \lim_{S \to \infty} |\rho(S)| = \lim_{S \to \infty} |\bar{\rho}(S)| \]

If the symmetric scattering amplitude dominates at high energies, we have (Eden, 1967),

\[ F(S,0) = C_2 \Lambda N \left[ \delta_N(-\Lambda S) \right]^{2\gamma} \]

And

Thus, the real part of the forward amplitude becomes positive for sufficiently high energies.

In the forward direction, we have

\[ \frac{d\sigma(S,0)}{dt} = \frac{\sigma_T^2}{16\pi} \left[ 1 + \rho^*(S) \right] \]

Using eq. (1) and the results given in eq. (7), we obtain

\[ \lim_{S \to \infty} \frac{d\sigma(S,0)}{dt} = \lim_{S \to \infty} \frac{d\sigma(S,0)}{dt} = \frac{C_1^2 (\Lambda N S)^{4\gamma}}{16\pi} \left[ 1 + \rho^*(S) \right] \]

If we define the diffraction width as (Logunov et al., 1973),

\[ \Delta(S) = \sigma_{el}(S) / \frac{d\sigma(S,0)}{dt} \]
we find that,

\[ \Delta(L) = C_n / (L-n)^r \]  

To obtain this result use was made of eqs. (5) and (9). Thus eq. (11) leads us to the conclusion that if the elastic and total cross sections are increasing with the same energy dependence, the slope of the diffraction peak must increase with energy.

RESULTS

Let us now take a look at the data (Amaldi et al., 1973; Bartenev et al., 1972). Certainly since the elastic and total cross sections are increasing with energy we must have energy behavior that is consistent with both eqs. (1) and (5).

However, the data are not over a sufficiently large interval such that \( \gamma \) can be uniquely determined. The values of \( \rho \) obtained are consistent with the real part of the forward scattering amplitude becoming positive at low energies (Amaldi et al., 1973). Results on the slope of the forward diffraction peak (Bartenev et al., 1972) show that it increases with energy and can be fit approximately by the form, \( b(S) = b_0 + b_1 \ln S \). However, the data could probably be fit with other forms for \( b(S) \). Since the results given in this note are really asymptotic results, we must be careful at this time of making detailed comparisons between theory and experiment.

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LITERATURE CITED


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