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PROPERTIES OF SCATTERING AMPLITUDES WHOSE PARTIAL WAVES DECREASE EXPONENTIALLY

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ABSTRACT

This paper presents the properties of elastic scattering amplitudes that have partial waves, $f_l(s)$, which decrease exponentially with l. The amplitude of this paper is a model for elastic scattering processes at asymptotic energies where all physical quantities transform as homogeneous functions of the appropriate dimension in the energy variable (Mickens, 1976).

INTRODUCTION

A method of determining the properties of an elastic scattering amplitude is to investigate the corresponding partial wave series. In this paper a study is made of the properties of scattering amplitudes whose partial waves decrease exponentially.

CALCULATIONS

Consider the elastic scattering of two scalar particles of equal mass M and denote the scattering amplitude by F(S,t). This amplitude has the following partial wave expansion (Eden, 1967),

$$(1)\quad F(S,t) = \frac{8\pi\sqrt{S}}{K} \sum_{l=0}^{\infty} (2l+1) f_l(S)\, P_l(\cos\theta),$$

where,

$$(2)\quad \begin{cases} S = 4(M^2+K^2), \\ t = -2K^2(1-\cos\theta), \end{cases}$$

and the unitarity condition is,

$$(3)\quad 0 \le |f_l|^2 \le \mathrm{Im} f_l \le 1.$$

The total cross section is given by the following expression (Eden, 1967),

$$(4)\quad \sigma_T(S) = \left[\frac{1}{2K\sqrt{S}}\right] \mathrm{Im}\, F(S,0).$$

The physical region corresponds to $S \geqq 4m^2$ and $-4k^2 \leqq t \leqq 0$.

The major assumption is that the partial waves, $f_l(s)$, decrease "exponentially" with l, i.e.,

$$(5)\quad f_l(S) = C h^l, \quad l = 0, 1, 2, \ldots,$$

where h(S) is a real valued function of S, satisfying in the physical region the condition

$$(6a)\quad 0 < h(S) < 1,$$

and C(S) is a complex function of S. Unitarity requires that,

$$(6b)\quad 0 \le |C(S)|^2 \le \mathrm{Im}\, C(S) \le 1.$$

Finally, $\mathrm{Re}C(S) \equiv d(S)$ and $\mathrm{Im}C(S) \equiv a(S)$ will be defined. Other than the conditions given by equations (6a) and (6b), no further restrictions will be placed on C(S) and h(S).

Substitution of equation (5) into equation (1) and using the fact that for $0 < h < 1$ (Hobson, 1931),

$$(7)\quad \sum_{l=0}^{\infty} (2l+1) h^l P_l(\cos\theta) = \frac{1-h^2}{\left[1-2h\cos\theta+h^2\right]^{3/2}},$$

the scattering amplitude is found to be,

$$(8)\quad F(S,t) = 8\pi\sqrt{\frac{S}{K^2}}\; \frac{(1-h^2)\,C}{\left[1-2h\cos\theta+h^2\right]^{3/2}}.$$

In terms of the variable t, the left-hand side of equation (8) may be written,

$$(9)\quad F(S,t) = \sqrt{\frac{S}{K^2}}\; \frac{\left[8\pi(1-h^2)C\right]}{\left(\frac{h}{2K^2}\right)^{3/2}\left[\left(\frac{2K^2}{h}\right)(1-h)^2 - t\right]^{3/2}}.$$

Note the amplitude, equation (9), has no zeroes in the t-physical region and, in addition, for fixed S, the amplitude is a monotonic decreasing function of t. These results imply that the differential cross section is a monotonic decreasing function of t with no zeroes.

It is easily seen that the phase, defined as the ratio of the real to the imaginary parts of the amplitude, is a function of only the energy variable S, i.e., it is independent of t,

$$(10)\quad \Phi(S,t) \equiv \frac{\mathrm{Re}\, F(S,t)}{\mathrm{Im}\, F(S,t)} = \frac{d(S)}{a(S)} = \bar{\Phi}(S).$$

From the unitarity condition, equation (6), the folowing bound on the phase is obtained,

$$(11)\quad |\Phi(S,t)| = |\bar{\Phi}(S)| \le 1.$$

Using equations (4) and (9), the total cross section may be calculated; it is,

$$(12)\quad \sigma_T(S) = \frac{16\pi\, a\,(1+h)}{(1-h)^2\,(S-4M^2)}$$

The requirements of equation (6) imply that the total cross section decreases essentially as the inverse of the energy variable S.

The diffraction width is defined as (Eden, 1967),

$$(13)\quad \Delta(S) \equiv \frac{d}{dt}\left\{\mathrm{Log}\; \mathrm{Im}\, F(S,t)\right\}\Big|_{t=0}.$$

It is easily calculated and has the value,

$$(14)\quad \Delta(S) = \left[\frac{h}{(1-h)^2}\right]\left[\frac{3}{S-4M^2}\right].$$

The differential cross section is given in terms of the amplitude F(S,t) by the expression (Eden, 1967),

$$(15)\quad \frac{d\sigma(S,t)}{dt} = \frac{|F(S,t)|^2}{16\pi S(S-4M^2)},$$

Substitution of equation (9) into equation (15) gives the following for the differential cross section,

$$(16)\quad \frac{d\sigma(S,t)}{dt} = \frac{16\pi|C|^2(1-h^2)^2}{(S-4M^2)^2\left[1-2h\cos\theta+h^2\right]^3}$$

The forward differential cross section is,

$$(17)\quad \frac{d\sigma(S,0)}{dt} = \frac{16\pi|C|^2(1+h)^2}{(S-4M^2)^2(1-h)^4}.$$

An easy calculation gives,

$$(18)\quad \sigma_{elastic}(S) = \left[\frac{4\pi}{S-4M^2}\right]\left[\frac{|C|^2}{h}\right] \times \left\{1 - \frac{(1-h^2)^2}{(1-h^2+2h)^2}\right\}.$$

SUMMARY

Let us summarize the above results: (i) The phase is a function only of S and is bounded in absolute value by one; (ii) The diffraction width, total and elastic cross sections all decrease with energy (essentially) as the inverse of the energy S; and (iii) The forward differential cross section decreases (essentially) as the S^{-2}. It is the exponential decrease of the partial waves which is responsible for the rapid decrease in various physical quantities.

It is of interest to note that the same properties come out of a consideration of scattering processes at asymptotic energies where we assume that all physical quantities transform as homogeneous functions of the appropriate dimensions in the energy variable (Mickens, 1976). This is equivalent to having the lack of any fundamental energy scale at very high energies.

Finally, it may be pointed out, that as a function of the variable t, the amplitude, as given by equation (9), has a branch point which is a function of S. Its location is,

$$(19)\quad t = \left[\frac{S-4M^2}{2h}\right](1-h)^2.$$

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