Religious Uncertainty and Anxiety Aroused in Eighth Grade Students. Stanley Jones, Tennessee Wesleyan College. It has been observed that religious questioning tends to cause anxiety. In this study is was hypothesized that the answering of questions about religion would raise anxiety, that the amount of uncertainty about religious questions would predict the amount of anxiety aroused, and that the males would show more anxiety than the females. Students in an 8th grade class were tested for base anxiety one morning and then retested the next day after taking a test on religious beliefs. A control group was tested and retested on anxiety but given a general information test instead of a test of religious beliefs. The Anxiety Differential Test of Husek and Alexander was used to measure anxiety. Anxiety raised significantly for males but not for females as a function of taking a test on religious beliefs. Amounts of uncertainty did not correlate with amount of increase in anxiety. Overall, males showed a significantly greater increase in anxiety than females.

The Use of Operant Conditioning Techniques with the Brain-Damaged Child. Harold Maready, Tennessee Wesleyan College. The purpose of this study was to demonstrate the use of operant conditioning techniques in an attempt to lengthen the attention span of a brain-damaged boy. This study attempted to lengthen the attention span by giving the subject a reinforcement (soda cracker) for a desired behavior. There were four main types of behavior desired: (1) inter-personal contact, (2) attention to the task, (3) reduction of autistic play activity, (4) reduction of attempts to leave the room. The results showed no significant difference in the inter-personal contact or the attention to the task. There was a significance in the amount of autistic play activity and in the attempts to leave the room.

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A NEW TROGLOBITIC TRICHONISCID ISOPOD OF THE GENUS *CAUCASONETHES*

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ABSTRACT

The new species, *Caucasonethes paynei,* from eastern Tennessee, is the third form of this genus to be described from eastern U.S. caves.

The trichoniscid isopod genus *Caucasonethes* Verhoeff has been known to include only two species, both troglobitic, in the eastern United States. These are *Caucasonethes henroti* (Vandel), from Gilley Cave, Spangler Cave and Gallohan Cave (Number one), all in Lee County, Virginia (Holsinger 1967), and *C. nicholasi* Vandel, from Columbia Caverns, Dickson County, Tennessee (Vandel 1965). This paper describes a third species on the basis of specimens collected from a cave in Anderson County, Tennessee, by Dr. Jerry A. Payne. Types are deposited in the United States National Museum.

Caucasonethes paynei, new species

Description: Generally similar to *C. henroti* (Vandel 1950) and *C. nicholasi* (Vandel 1965). The single male is 2.5 mm long and 0.75 mm wide; the eight females range in length 3.0 to 3.5 mm and in width 1.0 to 1.3 mm. Bodies of the alcohol-preserved specimens without pigment, white or yellowish-white in color.

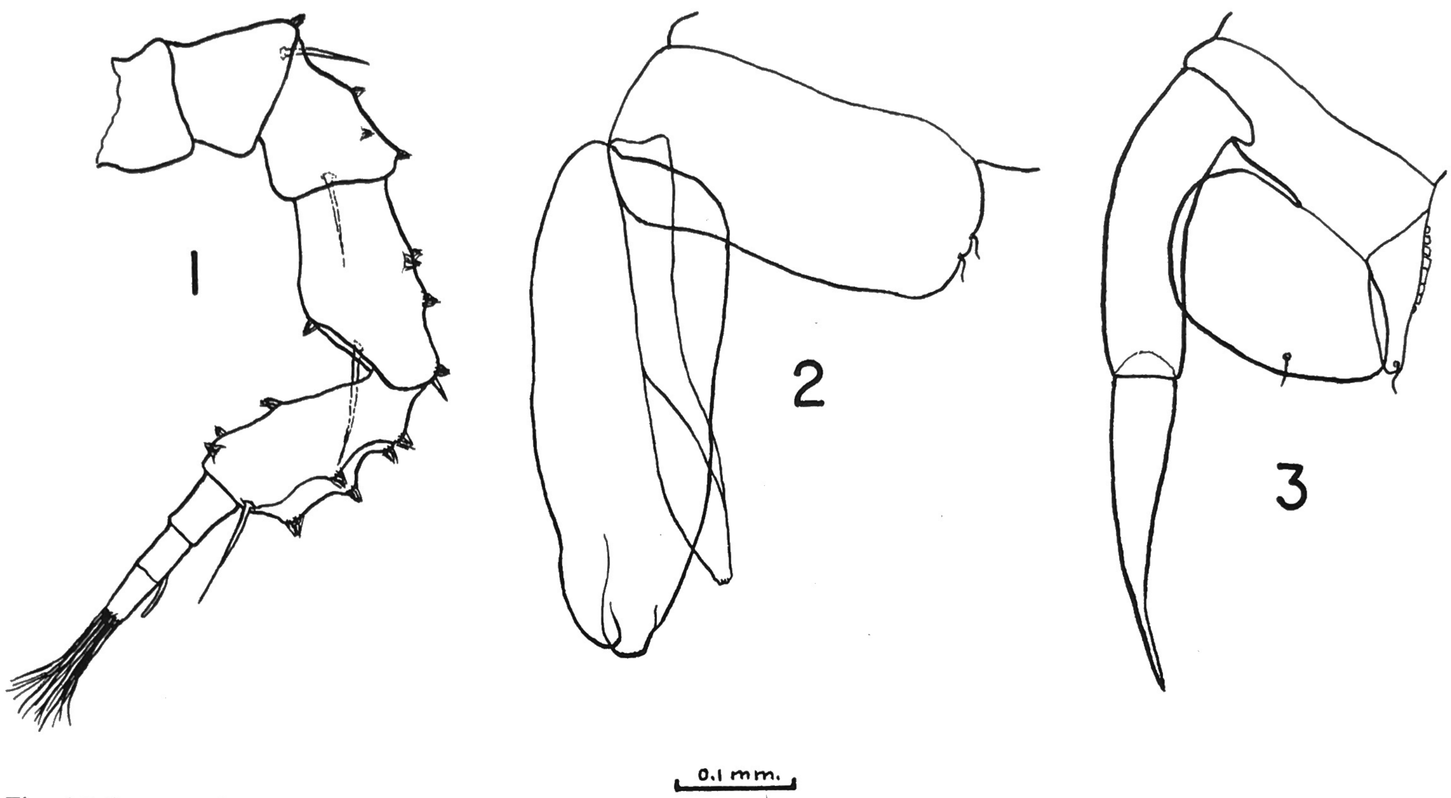
No eyes present.

Dorsal surfaces of body bearing conspicuous tubercles, which are arranged in four transverse rows on the head, in three rows on pereionite I, in two rows on pereionites II-V, in one or two rows on pereionite VI, and in a single row on pereionite VII; pleonites and telson essentially smooth.

Antennule small; terminal segment bearing four aesthetascs arranged in a row along its edge.

Antenna slightly stouter than that of *C. nicholasi* (Fig. 1). Outer surface of fourth segment with four tubercles bearing long scales, and outer surface of fifth segment with five similar tubercles. Flagellum of three distinct articles, the joint between the second and third apparently immovable; second article bearing a single aesthetasc.

No evident sexual dimorphism exhibited by the legs, but first and second pleopods of male modified as in other members of the genus. First pleopod of male as shown in Figure 2; endopodite elongate and expanded in the distal third; exopodite somewhat narrower than that of *C. nicholasi* and with shorter, more rounded terminal lobes. Second pleopod of male as shown in Figure 3; endopodite with the two segments about equal in length, the distal segment terminating in a fine



Figs. 1-3 *Caucasonethes paynei*, new species. 1. Antenna. 2. First pleopod of male. 3. Second pleopod of male.

point; exopodite rounded, nearly egg-shaped in outline.

Material: The type series consists of one male, eight females and one juvenile collected from Offutt's Cave, about 3.5 miles north of Clinton, Anderson County, Tennessee (Lat. 36°09′56″ N. Long. 84°07′52″ W.) by J. A. Payne on the following dates: 27 March 1965, 10 April 1965, and 11 July 1965.

In addition, two females and one juvenile are at hand, collected by Payne in Melton Hill Cave #1, about 7 miles south of Oak Ridge, in Roane County, Tennessee (Lat. 35°53′37″ N. Long 84°19′24″ W.), and about 20 miles southwest of Offutt's Cave. While these specimens are generally similar to those included in the type series of *C. paynei*, they differ from the latter in several respects, particularly in the size and distribution of the tubercles on the dorsum. In the absence of males from this cave, it is impossible to decide whether these are variants of *C. paynei* or are representatives of yet another species.

Remarks: Using the key to species of *Caucasonethes* provided by Vandel (1965, p. 368), *C. paynei* will run to the second alternative under B, that is to *C. nicholasi*. These two species can be distinguished by means of the following couplet:

Tubercles present on all pereionites; antenna with flagellum of three articles. *C. paynei*, new species.
Tubercles present only on pereionites I-IV; antenna with flagellum of two articles. *C. nicholasi* Vandel.

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ON THE ORDER AND TYPE OF ENTIRE FUNCTIONS

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Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be an entire function of order ζ, lower order λ, $(0 < \lambda \leq \zeta < \infty)$, type T and lower type t $(0 < t \leq T \leq \infty)$. $|a_n/a_{n+1}|$ be non-decreasing function of n for $n \geq n_0$. Then it is known that [1], [2],

$$\lim_{n\to\infty} \begin{matrix}\sup\\ \inf\end{matrix} \frac{n.\log n}{\log|a_n|^{-1}} = \begin{matrix}\zeta\\ \lambda\end{matrix},$$

and [3]

$$\lim_{n\to\infty} \begin{matrix}\sup\\ \inf\end{matrix} \frac{n.|a_n|^{\zeta/n}}{\zeta e} = \begin{matrix}T\\ t\end{matrix},$$

We now define Ratio-order "L" and Ratio-type "M" of f(z) as,

$$L = \frac{\zeta}{\lambda} \qquad \text{and} \qquad M = \frac{T}{t}$$

The object of this paper is to investigate the relationship between the orders and types of two or more entire functions and study the relations between the coefficients in the Taylor expansion of entire functions.

THEOREM 1:

Let $f_i(z) = \sum_{n=0}^{\infty} a_n^{(i)} z^n$, where i=1, 2, ..., k be k entire functions of orders ζ_i and lower orders λ_i $(0 < \lambda_i \leq \zeta_i < \infty)$, where i=1, 2, ...,k. $\left|a_n^{(i)}/a_{n+1}^{(i)}\right|$ be non-decreasing functions of n for $n \geq n_0$, where i=1, 2, ..., k.

Then the function

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

where,

$$k.\log(|a_n|^{-1})^{-1} \sim \sum_{i=1}^{k} (\log|a_n^{(i)}|^{-1})^{-1} \tag{1}$$

is an entire function of order ζ and lower order λ, such that

$$\sum_{i=1}^{k-1} \lambda_i \leq (k\lambda-\lambda_k, k\zeta-\zeta_k) \leq \sum_{i=1}^{k-1} \zeta_i \tag{2}$$

COROLLARY: I) If any k functions out of k+1 functions $f(z), f_1(z), f_2(z), \ldots, f_k(z)$ are regular then all the k+1 functions are regular and

$$\zeta k = \sum_{i=1}^{k} \zeta_i$$

II)

$$L \leq \frac{\sum_{i=1}^{k} L_i \lambda_i}{\sum_{i=1}^{k} \lambda_i}$$

where $L, L_1, L_2, \ldots L_k$ are Ratio-orders of $f(z), f_1(z), f_2(z), \ldots, f_k(z)$ respectively.

THEOREM 2:

Let $f_i(z) = \sum_{n=0}^{\infty} a_n^{(i)} z^n$, where .,k be k entire functions of orders ζ_i, lower orders λ_i, types T_i and lower types t_i, $(0 < \lambda_i \leq \zeta_i < \infty$, $0 < t_i \leq T_i < \infty)$, $i=1,2,\ldots,k$. $\left|a_n^{(i)}/a_{n+1}^{(i)}\right|$ be non decreasing functions of n for $n \geq n_o$. $i=1,2,\ldots,k$.

Then the function

$$f(z) = \sum_{n=0}^{\infty} a_n z^n$$

where,

$$\log|a_n|^{-1} \sim \prod_{i=1}^{k} \left(\log|a_n^{(i)}|^{-1}\right)^{m_i}$$

$(0 < m_i < 1, \sum_{i=1}^{k} m_i = 1, i=1,2,\ldots,k)$

is an entire function of order ζ, lower order λ, type T and lower type t, such that

$$\prod_{i=1}^{k-1} \lambda_i^{m_i} \leq \left(\frac{\lambda}{\lambda_k^{m_k}}, \frac{\zeta}{\zeta_k^{m_k}}\right) \leq \prod_{i=1}^{k-1} \zeta_i^{m_i}$$

and

$$\prod_{i=1}^{k-1} t_i^{m_i} \leq \left(\frac{t}{t_k^{m_k}}, \frac{T}{T_k^{m_k}}\right) \leq \prod_{i=1}^{k-1} T_i^{m_i}$$

COROLLARY: I) If any k functions out of k+1 functions $f(z), f_1(z), f_2(z), \ldots, f_k(z)$ are of regular growth or perfectly regular growth then all the k+1 functions are of regular growth or perfectly regular growth respectively.

II)

$$L \leq \prod_{i=1}^{k} L_i^{m_i}$$

and

$$M \leq \prod_{i=1}^{k} M_i^{m_i}$$

where $L, L_1, L_2, \ldots, L_k$ and $M, M_1, M_2, \ldots, M_k$ are Ratio-orders and Ratio-types of $f(z), f_1(z), f_2(z), \ldots, f_k(z)$ respectively.

These results improve the results of Pavan-kumar Kamthan [4].

PROOF OF THEOREM 1:

We know [5],

$$\lim_{n\to\infty} {}^{\sup}_{\inf} \frac{n.\log n}{\log\left|a_n^{(i)}\right|^{-1}} = \frac{\zeta_i}{\lambda_i}$$

So,

$$\frac{n.\log n}{\log\left|a_n^{(i)}\right|^{-1}} < \zeta_i + \epsilon \text{ for } n \geq n_0 \qquad (3)$$

and

$$\frac{n.\log n}{\log\left|a_n^{(i)}\right|^{-1}} < \lambda_i + \epsilon \text{ for a sequence of values of } n\to\infty \qquad (4)$$

Taking $i=1,2,\ldots,k-1,k$ in (3) and adding, we get

$$\sum_{i=1}^{k} \frac{n.\log n}{\log\left|a_n^{(i)}\right|^{-1}} < \sum_{i=1}^{k} \zeta_i + \epsilon$$

So, using (1) we get

$$\frac{kn.\log n}{\log|a_n|^{-1}} < \sum_{i=1}^{k} \zeta_i + \epsilon.$$

Taking limit,

$$k\zeta \leq \sum_{i=1}^{k} \zeta_i$$

i.e.

$$k\zeta - \zeta_k \leq \sum_{i=1}^{k-1} \zeta_i$$

which proves the part of R.H.S. of (2).

We now prove the other part of R.H.S. of (2).

Taking $i=1,2,\ldots,k-1$ in (3) and $i=k$ in (4), and adding and using (1) we get,

$$\frac{kn.\log n}{\log|a_n|^{-1}} < \sum_{i=1}^{k-1} \zeta_i + \lambda_k + \epsilon$$

Taking limit,

$$k\lambda \leq \sum_{i=1}^{k-1} \zeta_i + \lambda_k$$

i.e.

$$k\lambda - \lambda_k \leq \sum_{i=1}^{k-1} \zeta_i$$

which proves the other part of R.H.S. of (2). Similarly L.H.S. follows.

PROOF OF COROLLARY: I) We omit the proof.

II) We know,

$$\zeta k \leq \sum_{i=1}^{k} \zeta_i$$

i.e.

$$k\lambda L \leq \sum_{i=1}^{k} \lambda_i L_i$$

i.e.

$$L \leq \frac{\sum_{i=1}^{k} \lambda_i L_i}{k\lambda}, \text{ but } \sum_{i=1}^{k} \lambda_i \leq k\lambda.$$

So,

$$L \leq \frac{\sum_{i=1}^{k} \lambda_i L_i}{\sum_{i=1}^{k} \lambda_i}$$

We omit the proof of theorem 2.

SUMMARY

Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be an entire function of order ζ and lower order λ, $(0 < \lambda \leq \zeta < \infty)$, type T and lower type t $(0 < t \leq T < \infty)$. $|a_n/a_{n+1}|$ be non-decreasing function or n for $n \geq n_0$. Then it is known that,

$$\lim_{n\to\infty} {}^{\sup}_{\inf} \frac{n.\log n}{\log |a_n|^{-1}} = {}^{\zeta}_{\lambda},$$

and

$$\lim_{n\to\infty} {}^{\sup}_{\inf} n.|a_n|^{\zeta/n} = {}^{e\,\zeta\,T}_{e\,\zeta\,t},$$

In this article we investigate the relationships between the orders, types, ratio-orders and ratio-types of two or more entire functions, where, "ratio-order" and "ratio-type" are defined as follows:

$$\text{Ratio-order } L = \frac{\zeta}{\lambda} \quad \text{and} \quad \text{Ratio-type } M = \frac{T}{t}$$

For example,

$$\sum_{i=1}^{k-1} \lambda_i \leq (k\lambda - \lambda_k, k\zeta - \zeta_k) \leq \sum_{i=1}^{k-1} \zeta_i$$

if

$$k.\log(|a_n|^{-1})^{-1} \sim \sum_{i=1}^{k} (\log |a_n^{(i)}|^{-1})^{-1}$$

where ζ_i and λ_i are orders and lower orders of $f_i(z) = \sum_{n=0}^{\infty} a_n^{(i)} z^n$, $(i=1,2,\ldots,k)$.

Some more results of similar nature are proved in this article.

Finally I wish to thank Prof. Dr. H. Hornich for his kind interest and helpful criticism and Govt. of Austria for awarding me a scholarship.

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