(10) $N_1^*(a) = N_2^*(a) = \begin{cases} 6 & (\Psi(a) = -1, \Psi(-1) = 1), \\ 0 & \text{(otherwise)}. \end{cases}$

Case 2 (Oblique planes). We consider the general oblique plane

(11) $b = x_1 + \beta_2 x_2 + \beta_3 x_3 (\beta_2\beta_3 \neq 0)$.

Eliminating X_1 between (1) and (11), it follows that (1) and (11) have a common point if and only if the equation

(12) $a - b^2 = (c_2 x_2^2 + 2\beta_2\beta_3 x_2 x_3 + c_3 x_3^2) - 2b(\beta_2 x_2 + \beta_3 x_3)$,

where $c_2 = \beta_2^2 + 1, c_3 = \beta_3^2 + 1$, is solvable in F. The quadratic form enclosed in parentheses in (12) has determinant $\Delta = 4(\beta_2^2\beta_3^2 - c_2c_3)$. If $\Delta \neq 0$, this form is congruent to a form, $\alpha_1 x_1^2 + \alpha_2 x_2^2$ $(\alpha_1\alpha_2 \neq 0)$, by Lemma 3. Hence by the corollary to Lemma 2, it follows as in Case 2 that (12) is solvable.

Suppose then that $\Delta = 0$, in which case it results that $c_2 \neq 0, c_3 \neq 0$. A simple computation shows that (12) assumes the form,

(13)

$$\frac{ac_2 - b_2}{c_2^2} = \left(X - \frac{b\beta_2}{c_2}\right)^2 + \frac{2bx_3}{c_2\beta_3}\left(X = x_2 + \frac{c_3x_3}{\beta_2\beta_3}\right)$$

If $b \neq 0$, then (13) can be solved with X assigned arbitrarily. Assume therefore that $b = 0$. Then (13) is insolvable if and only if ac_2 is a non-square of F.

We have thus shown that (1) and (11) fail to have a common point if and only if $\Delta = b = 0, \Psi(ac_2) = -1$, or what is the same, if and only if

(14) $b = 0, \Psi(-a) = -1, \beta_2^2 + \beta_3^2 = -1$.

But by Lemma 2, the system

$$\beta_2^2 + \beta_3^2 = -1, \beta_2 \neq 0, \beta_3 \neq 0,$$

has $q-5$ or $q+1$ solutions according as -1 is a square or a nonsquare of F. Therefore, if $N_3^*(a)$ denotes the number of homogeneous, oblique planes contained in the complement of (1), we have

(15)

$$N_3(a) = N_3^*(a) = \begin{cases} q+1 & (\Psi(-a) = \Psi(-1) = -1) \\ q-5 & (\Psi(-a) = -1, \Psi(-1) = 1) \\ 0 & \text{(otherwise)}. \end{cases}$$

Since $N_1(a) = N_1^*(a) + N_3(a), N_2(a) = N_2^*(a) + N_3^*(a)$, (3) follows on the basis of (10) and (15). The conclusion (4) results from Lemma 1 and (15), on recalling that -1 is a square in P if and only if $p \equiv 1 \pmod 4$.

Corollary 1. *The only cases in which* $N_1(a) > 0$, $N_3(a) = 0$, *occur when* $q = 5, a = \pm 2$.

Corollary 2. *The case* $q = 3, a = 1$ *is the only one in which the complement of* (1) *contains all oblique, homogeneous planes of* S_3.

By Lemma 3 we obtain from (3) our principal result:

Theorem 2. *Let* Q *denote a ternary quadratic form over* F *of determinant* $\Delta \neq 0$, *and let* α *be an arbitrary element of* F. *All planes of* S_3 *contained in the complement of the quadric,* $Q = \alpha$, *are necessarily homogeneous (that is, subspaces); the number* $N(\alpha)$ *of such planes is*

$$N(\alpha) = \begin{cases} q+1 & \textit{if } \Psi(-\alpha\Delta) = -1, \\ 0 & \textit{otherwise}. \end{cases}$$

LITERATURE CITED

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THE EIGHTEENTH TENNESSEE SCIENCE TALENT SEARCH

and

THE TWENTY-FIRST TENNESSEE JUNIOR ACADEMY OF SCIENCE

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The 1963 programs of the Tennessee Science Talent Search and the Tennessee Junior Academy of Science have given great impetus to the development of Junior Scientists in the state. The two programs, sponsored by the Tennessee Academy of Science with the cooperation and assistance of the science and mathematics teachers of the secondary schools, are directed by James L. Major, Clarksville High School, Clarksville, and Myron S. McCay, University of Chattanooga, Chattanooga, respectively.

To qualify for competition in the Tennessee Science Talent Search, entrants must be high school seniors, must pass a comprehensive examination, and report on an original science project. This year 36 students in 24 schools in 16 communities were selected as state winners. The winners came from all regions of the state, a fact most gratifying to the Talent Search Committee, one of whose stated aims is to have all schools in Tennessee participate in the program. The Academy forwarded to a large number of colleges the names of talented science students to be considered for admission, scholarships, for assistantships in science laboratories and similar positions.

The success of a continuing and productive Talent Search depends, in a large measure, upon a supporting

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