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# SOME NOTES ON GEOMETRY AS A SCHOOL SUBJECT

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For more than two hundred years after the first college was founded in the United States, Euclidian geometry was essentially a college subject. Harvard, established in 1636, offered geometry of a practical sort in the senior year. Yale followed Harvard's lead, and in these schools it was more than a century before geometry was taught in the freshman year. As other colleges and universities were established, Harvard's program or a similar one was followed.

Early schools manifested little interest in geometry as such, usually uniting it with astronomy, natural philosophy, navigation or surveying. Printed texts were not used until 1726, when Ward's *Young Mathematician's Guide*, a text not strict in logic, was introduced (Stamper, 1909, p. 97). Euclid, introduced into Harvard in the decade following 1730 and subsequently adopted by many other colleges and universities, seems to have been studied with little attention to its logical structure. A story by Samuel G. Brown, a graduate of Dartmouth, reflects an early teaching practice: "I remember hearing one of the older graduates say that the first lesson in his class in geometry was twenty pages of Euclid, the instructor remarking that he should require only the captions of the propositions, but if any doubted the truth of them, he might read the demonstrations, though for his part, his mind was perfectly satisfied" (Cajori, 1890, p. 74).

Toward the close of the eighteenth century, schools began to show greater interest in geometry. Euclid was taught more generally and its teaching became more rigorous. The influence of English books was strong, and reliance upon the textbook became pronounced. The first American edition of Euclid appears to have been an English translation published in Worcester in 1784 (Cajori, 1890, p. 55), but it was Robert Simson's Euclid, published in 1803, that made the first real imprint on the textbooks in this country. His book not only enjoyed great popularity but was the foundation for many texts for the next hundred years. Simson's book was gradually displaced by an edition of Euclid by John Playfair, and to a lesser extent, by an edition by John Allen. Playfair had departed from the standard Euclid: he had simplified certain sections, employed greater use of algebra, altered definitions, and introduced some new propositions. To the standard Euclid, Allen added trigonometry, conic sections, and elements of natural philosophy as far as it was related to astronomy. One object of his book was to make available information necessary for a better understanding of Newton's work. The books by Playfair and Allen indicate the British idea of the time ". . . to revise, simplify, and enrich the text of Euclid" (Cajori, 1897, p.

286). It was during this period that the value of geometry as a disciplinary subject became firmly established.

French geometries followed the influx of the English geometries into this country. Karpinski (1949, p. 11) found that the most popular of the French texts was Legendre's, first published in the United States in 1810. It differed from the English books in general use: Legendre had formulated a new sequential order, assumed a correspondence between a line segment and a number, made use of hypothetical constructions, and extended the use of algebra in proving theorems. Also he had simplified the subject matter. The preface of his text was probably the first to state the dual purpose of geometrical study: "Geometry is not studied merely for the facts it teaches, but because it disciplines the untrained intellect and conducts the untaught mind to the temple of truth" (Legendre, 1828, p. vi).

Legendre's text had a long history in this country. The most popular edition was by Charles Davies, who published a translation by Thomas Carlyle, edited by David Brewster. "By 1850 Davies had attached his name to Legendre's eleven times by way of Brewster and the labor of Carlyle. This long familiarity finally reconciled Davies to using his own name with the translation without mentioning either Brewster or Carlyle" (Karpinski, 1949, p. 11). The book remained in active circulation for almost a century, displacing the English texts in many schools. The French influence tended to make geometry more practical, but the English influence emphasizing logical rigor remained strong (Stamper, 1909, p. 100). Most texts that followed Legendre's used his sequential order.

The influence of German geometries was not pronounced but its presence was shown by the use of the text, *An Introduction to Geometry and the Science of Form,* prepared from the most approved Prussian textbooks. The book was published anonymously in this country in 1843. In an introductory note to the book, Benjamin Peirce (1843, p. xix) stated that it was elementary and reflected the practical aspects of geometry. Its logic was not Euclidian. When Harvard placed geometry on the list of subjects required for entrance, it was on the contents of this book that candidates were examined (Broome, 1903, p. 45).

During the second quarter of the nineteenth century, geometries by American authors appeared. The influence of both English and French books was evident; in general American authors abandoned the standard Euclid, but modified Legendre's text to make it resemble Euclid as much as possible. Of these early geometries it has been said (Smith and Ginsburg, 1900, p. 82): "Few have done much to give real insight into the subject or to show the best of foreign influence." But gradually American texts were adopted. The following five were among the leaders in the field.

Geometries by Benjamin Peirce and Elias Loomis appeared in 1837 and 1847 respectively. Peirce's text was never widely popular. Some features of his book, adversely criticized in his day but long since accepted, were the recognition of direction as a fundamental

idea and the introduction of new assumptions to simplify some of the longer demonstrations of Euclid (Cajori, 1890. p. 135). Loomis's book was written in simple style, within the grasp of the student with ordinary ability. It was well suited to teaching and it became very popular.

William Chauvenet, an author of college textbooks, was ". . . probably the first to advance materially the study of elementary geometry and to reflect this influence in a notable way" (Smith and Ginsburg, 1900, p. 82). His book, published in 1870, was outstanding in its treatment of Euclidian geometry and contained a noteworthy section on modern geometry. It was recognized by mathematicians of the day as superior to contemporaneous texts and was used in many of the best schools. Chauvenet's text represented a greater departure from Euclid than did most early American texts. His influence was noticeable in books by later authors.

Edward Olney's first geometry was published in 1872. The greater amount of space devoted to informal geometry and the increased number of topics introduced in concrete settings indicate the book may have been in advance of its time. It was not readily accepted and the publishers advised a book of the more traditional type. Olney's later texts were revised to meet the demands of the day.

In 1878 a new textbook destined to affect all texts to follow was written by G. A. Wentworth. It contained a substantial increase in the number of exercises and was the first book to employ the unit page. It was the first truly successful rival of Davies' Legendre. By 1895 it was used in almost 75 percent of the schools studied by Stout (Stout, 1921, p. 122).

Until 1821 secondary school education, the purpose of which was to prepare for college and to train for life, was centered in the Latin Grammar School and the Academy. Brown (1910, p. 133) pointed out that in the college preparatory schools geometry had a regular place in the curriculum, but in the non-college preparatory schools, if taught at all, it was united with another subject. When the first high school was founded in 1821, geometry was included in the program of studies, and as more high schools were established, geometry was a part of many of their programs. In 1844, Harvard made geometry a requirement for entrance (Broome, 1903, p. 19). Soon other colleges and universities placed it upon their required lists, and between 1850 and 1875 it took a definite place in the secondary school curriculum (Smith, 1914, p. 51). For some time, however, it continued to be taught in many colleges. The foregoing texts were studied in both secondary schools and colleges.

As high schools grew, demands were placed upon them by business and industry and by colleges and universities. The problem of meeting these demands was presented at the meeting of the National Council of Education in 1891. Three years later, the Committee of Ten under the auspices of the National Education Association published a report. Since that date there have been many committees to publish reports concerned with the place and function of geometry. Most of these committees have worked under the separate or joint sponsorship

of three organizations: The Central Association of Science and Mathematics Teachers, The Mathematical Association of America, and the National Council of Teachers of Mathematics.

The Committee of Ten, reporting in 1894, assumed the disciplinary value of geometry and recommended greater rigor in demonstration. It urged that the teaching of concrete geometry begin in the primary schools and be continued through grade eight. Demonstrative geometry should then begin after one year of algebra and be carried on by the side of algebra for the next two years. The Committee (1894, p. 115) "opposed slavish dependence upon the textbook" and favored the practice of those colleges which called for original demonstrations on entrance examinations.

At the same time that improvements in geometry courses were being sought in the United States, John Perry of England (Perry, 1902), J. Tannery of France (Central Association of Science and Mathematics Teachers, 1950), and Felix Klein of Germany (Commission on Secondary School Curriculum, 1940) were leaders in similar movements in their respective countries. These men opposed strict adherence to Euclid and urged the axiomatic treatment of many theorems. More concreteness in geometry, closer correlation with science, and experimental work, as testing rules for areas, and solving triangles by scale drawings, were recommended. Graphical methods and the use of trigonometric functions were advised. This revised course was to allow for an earlier introduction of higher mathematics. In this country E. H. Moore (1926, p. 49) of the University of Chicago advocated proposals similar to those of Perry, Tannery, and Klein. Courses based on these proposals were attempted, but did not gain favor.

The influence of the reform groups was reflected in the report of the Committee of Fifteen, published in 1912. The Committee recognized the struggle between the formalists and the utilitarians but assumed the extreme position of neither, recommending that the logical structure of geometry be retained but that more attention be given to the practical value of the subject. The Committee favored a reduction in the number of required proofs of propositions and advised less attention to difficult abstract applications. Emphasis was placed upon simple exercises. All reports subsequent to 1912 have placed greater emphasis upon the utilitarian value of geometry and upon concreteness in its teaching.

In 1908 the International Congress of Mathematicians at Rome created the International Commission on the Teaching of Mathematics. The American members of the Commission directed four studies and reported between 1911 and 1918. In 1911 the Commission pointed out that as geometry had moved from the university to the secondary school, it had undergone little change; courses had not been revised to meet the needs of colleges nor examined to determine their fitness for later scientific work. This lack of change was attributed largely to the invulnerability of the sequential order of the text. The Commission (1912) found that the simultaneous teaching of algebra and geometry recommended by the Committee of Ten was practiced

in almost no schools in this country though it had been long tried in many other countries and was considered decidedly preferable.

The findings of this Commission and the generally unfavorable comparison of secondary school mathematics in the United States with that of foreign countries led to the appointment of the National Committee on Mathematical Requirements, which reported in 1923. This report defined the aims of instruction in broad outlines. Disciplinary values were related to general characteristics as attitudes, ideas, and ideals; utilitarian and cultural values were concerned with direct usefulness of geometrical facts and processes and with the development of appreciation of beauty in geometrical forms. A more flexible course and the correlation of geometry with other subjects were recommended. Suggestions for improvement in the college entrance examinations were presented to the Board with the hope of ending the circle of selecting a textbook conforming to the examination and adhering strictly to the textbook to prepare the student for the examination. In consonance with the suggestions of the Committee, the Board defined fewer theorems to be proved and allowed greater flexibility in quoting propositions.

From 1940 to 1944 three reports including sections on geometry were published. They were prepared by the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics (1940), the Commission on Secondary School Curriculum sponsored by the Progressive Education Association (1940), and the Commission on Post-War Plans created by the National Council of Teachers of Mathematics (1944). These reports stressed the need for a more comprehensive view of education and were concerned with relating geometry to needs and abilities. Recognition of the significance of geometry in human affairs, the application of deductive reasoning to life situations, and the establishment of a basis for inductive reasoning were recognized as important elements in the course.

The examinations of the College Entrance Examination Board reflect in part the influence of the committees' recommendations, showing the change from emphasis on formal discipline and unrelated segments of subject matter to an attempt to correlate geometry with other subjects. The definitions of the requirements adopted in 1903 continued in use until 1924 (College Entrance Examination Board, 1903-1924) with the one significant change concerning the propositions to be proved or quoted. The definitions adopted in 1923 were in use until 1942, when definitions were abandoned by the board (College Entrance Examination Board, 1924-1942). This move was regarded by many as a removal of restrictions on the content of courses, but some secondary schools and colleges have since expressed the need for some criterion upon which ". . . to base the judgment of preparation." (College Entrance Examination Board, 1948, p. 61).

It is interesting to note that geometry, a college subject for over two hundred years, had become, by 1875, a subject of the secondary school. In 1911 the International Commission reported that plane

geometry was taught in all public high schools and solid geometry in many. By 1920 courses in solid geometry were being discontinued. Courses combining plane and solid into a single course were advocated, and in 1924 the College Board agreed to offer an examination in the combined course. It did not prove popular, and in 1933, the examination was discontinued. At the present time neither plane nor solid geometry is offered in many high schools, and it is again taught in a number of colleges.

Since the report of the Committee of Ten in 1894, textbooks have attempted to conform to the recommendations of the committees, and authors have been aware of movements for reform. Many texts have been enriched by topics from algebra, trigonometry, analytic and modern geometry, and spherical trigonometry. A more interesting and rounded course is made possible by the inclusion of pictures, graphs, and other applications of geometric concepts. Many recent textbooks include topics to show the application of geometric reasoning to life situations and include problems designed to establish habits of critical thinking. It appears that more emphasis may be placed upon this phase of geometry, providing a pattern for logical thought and emphasizing the importance of definitions and assumptions accepted in everyday living.

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